Adaptive Biological Growth -

Introduction to Shape (CAO) & Topology (SKO) Optimization Derived from the Growing of Trees

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Summary:

Optimization of mechanical components becomes more and more popular in the Finite Element (FE) community and the daily practice of the design procedures.
The author of the book *Design in Nature: Learning from Trees* Prof. Mattheck now introduces what he calls the *Principle of Constant Stresses* derived from analogies observed in the growth of trees. He found that the trees adjust their growth in a fashion that the stresses on the surface are equally distributed. Stress peaks that occur will be reduced by a stress proportional growth in that area. Stimulating that 'growth' in a mechanical component, he therefore 'heats' the structure in areas of non-admissible stresses and lets the surface expand. He calls this way of Shape Optimization *CAO* (*Computer Aided Optimization*). The structures obtained from this approach do not show any significant stress peaks and therefore have no prescribed point of failure.
In addition, Mattheck introduces a method for the initial design that is needed for the above-described procedure. He observed that in nature, all unnecessary ballast is avoided and that material decays where it is no longer needed. He introduced what he called the *Soft Kill Option* (*SKO*). By varying the young's modulus in a structure, he rewards the ones that carry more of the load by increasing the young's modulus and punishes the elements at lower stress states by decreasing their respective young's modulus. By this, the 'lazy' elements increasingly withdraw themselves from carrying the load and once they do not contribute significantly, he purges them from the set of elements.
Applying both methods leads to a lightweight design that is cost efficient and durable.

Overview

Over time, various optimization strategies have been developed and they suit more or less the procedures used in FE programs and their environments. In the 'early days' of optimization technologies, parametric methods were used. This pure mathematical approach solves for the minimum of a cost function (optimisation goal). It requires a definition of optimization variables and then solves a system of linear dependent or independent equations. This method asks for a set of attributes or design variables (geometric, material etc.) to be changed within the optimization procedure. Disadvantage of this method is that the user has to have a thorough understanding of the structural behaviour and an 'idea' of what to optimise. That means that the input already defines the possible changes made to the initial design.

Closely related to the parametric strategies is the field of design sensitivities. As in the parametric optimization, the influence of the change of a parameter to the cost function is analysed. From the interrelationship of those variations, one can derive predictions for the response of the structure and therefore improve the 'next guess'.

In this paper, only two methods of non-parametric strategies are presented. There is no proof that they will achieve an optimal design but experience has shown that the application of the straightforward methods will result in lighter and durable structures. For the practitioner working on the design of a component it is important to know how to improve the performance. To him it is not so important whether the method is based on analogies or on mathematical-physical derivation.

It should also be mentioned that a 'true' optimum is hardly achieved. In a mathematical sense, there usually exist multiple relative and at least one global optimum. Moreover, the optimum with respect to its individual definition should maybe even be avoided. It should be remembered that due to statistical deviations and uncertainties the optimal design might fail if any of the assumptions is not met in the 'real' structure. Imperfections or loading conditions in combinations not foreseen might result in a total failure of the structure. In this sense, optimization can be regarded as the significant improvement over the initial design.

Observing the growth of other 'components' in nature such as bones, horns, and thorns also shows a possible reduction of material in areas of lower stresses.

The two methods proposed in this paper are well suited for the programming using standard FE programs. All that is needed is the access via an interface to the element topology and properties, stress results and the program's capability to calculate deflections from a temperature loading.

CAO - Computer Aided Optimization

Observing the growth of trees, Mattheck found two different kinds of possible adaptation to the change in load bearing. He found species that develop 'compression wood' and those developing 'tension wood'. In the first case, a tree trunk under bending will grow in a fashion that the side under primary compression will have broader annual rings where the latter develops those broader rings on the side of primary tension.

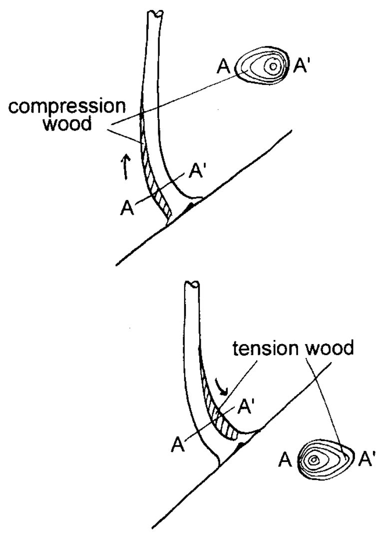


Figure 1 : Compression and Tension wood /1/

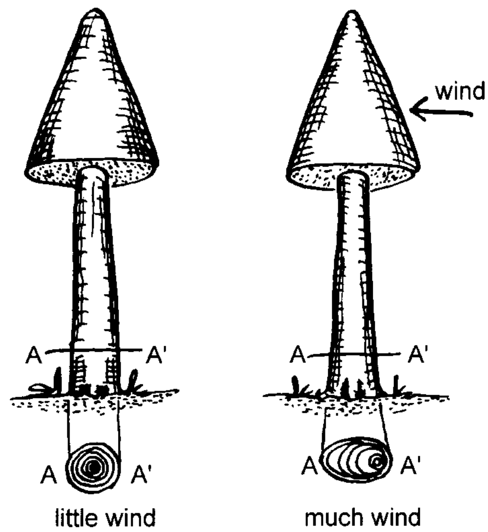


Figure 2 : Adaptation to Wind /1/

He concluded from this and the comparison with FE analyses of such systems that there is a growth accordingly to the side or area of higher stresses in a cross section and if you take both effects, compression and tension, you will come to an empirical rule that he transferred to the optimization scheme he calls CAO – Computer Aided Optimization.

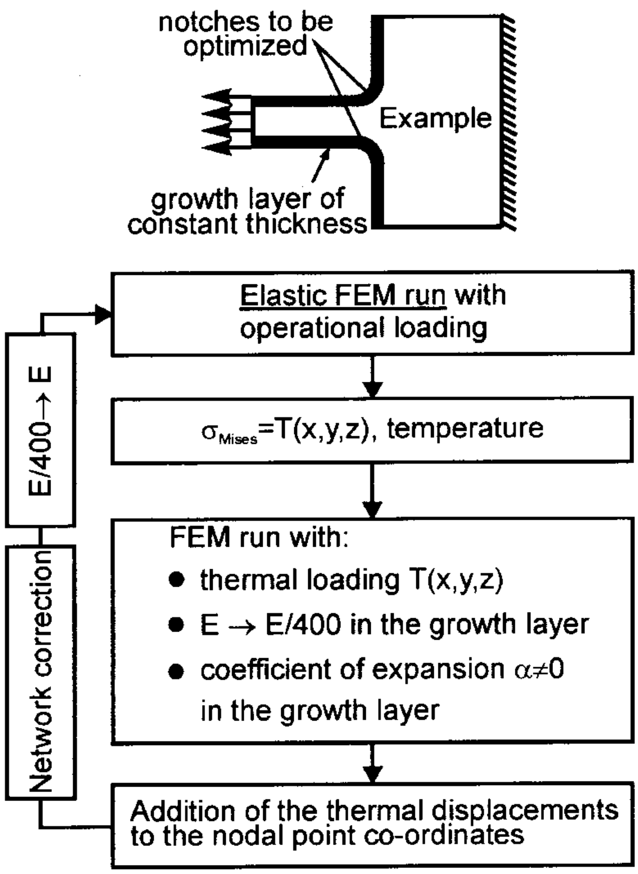


Figure 4 : CAO Iterative Procedure /1/

The underlying idea was to detect surface areas with high stresses (notches) at first. Now these areas have to be 'stimulated' to grow accordingly to the magnitude of unwanted stresses. The trick now is to heat a surface layer of almost equal thickness and 'freeze' the rest of the structure. To have an adequate magnitude in growth a reduction of the Young's modulus E to 1/400 of the initial value for the growth layer was chosen which makes the surface 'soft' to expand – or shrink. Now a temperature field is applied to the FE model and the respective deflections are determined. The initial nodal coordinates are moved to the locations they have reached under thermal expansion and this forms the 'updated' structure, which will be analysed under the initial loading.

If the notch stresses persist, the previous steps will be repeated until admissible stresses are reached.

Various analyses have shown that the final shapes will be close to the ones observed in nature and that the notch stresses 'disappear'. Mattheck defines his '*Axiom of Uniform Stresses*' by defining that the '*natural and adaptively grown notches do not cause notch stresses as long as they are properly loaded!*' .

As can easily be shown the comment on 'properly loaded' notches refers to the fact that an optimization will always be 'optimal' referring to the design criteria and parameters defined for the optimization procedure. If the optimized structure is faced with

In nature, the trees will adjust to the load that they most frequently face. It would be inefficient for those natural structures if they could bear any possible load and that means that in cases of e.g. exceptional storms trees might as well fail.

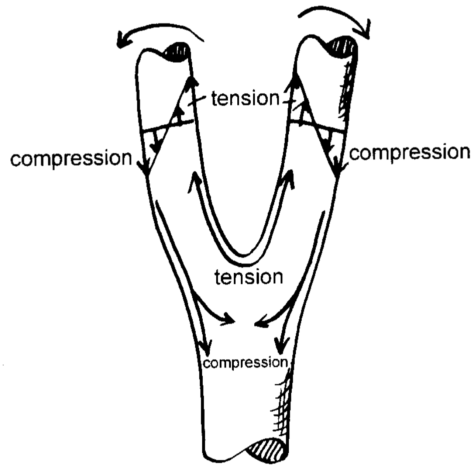


Figure 5 : Tension Fork /1/

Mattheck in his book /1/ gives various examples of shapes observed at trees and the according mechanical abstraction for the analysis. He shows the simply approach one could have using simple geometric elements such as rectangles, circles and arches and the shape that was determined applying the above stipulated procedure (fig. 4).

The example of a tree having two trunks in the upper and only one in the lower part as shown in fig. 5 gives the primary mechanical effects in this natural structure. Normal forces and bending due to the eccentricities cause high stresses in the middle of the fork.

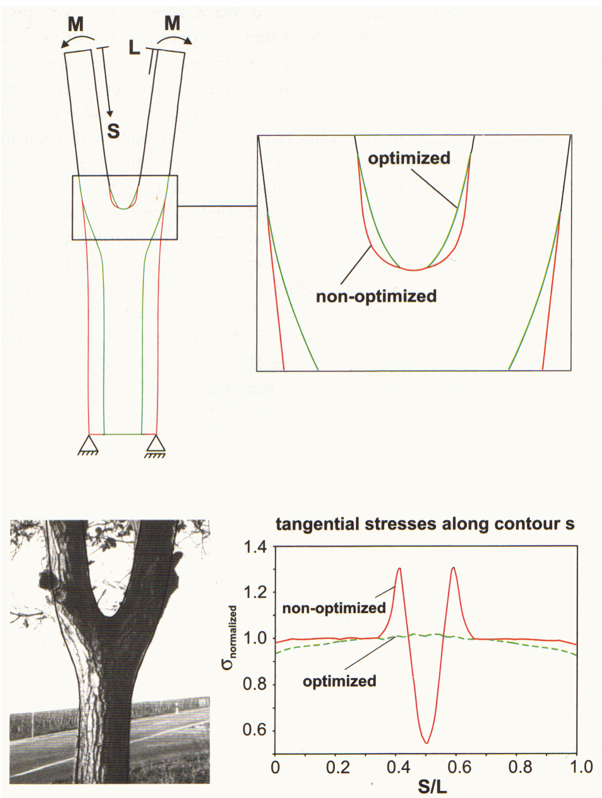


Figure 6 :
CAO Optimization of a Tension Fork /1/

Figure 6 shows the non-optimized shape using straight edges and a circular 'notch'. The stresses for this initial design are about 1.3 times the mean value along this edge.

Using the above-described algorithm the shape will be close to the one observed at a tree and will now have about the same uniform magnitude along this edge !

This effect is typical for a variety of assemblies as shown in figures 7, 8, and 9. Mattheck compares the abstract systems found in nature with those obtained using the CAO procedure.

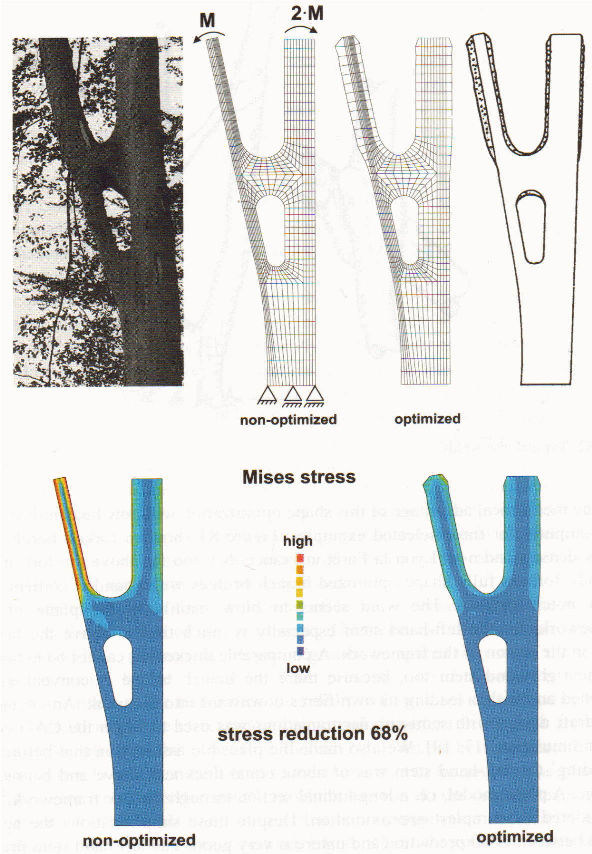


Figure 7 : Tree Framework /1/

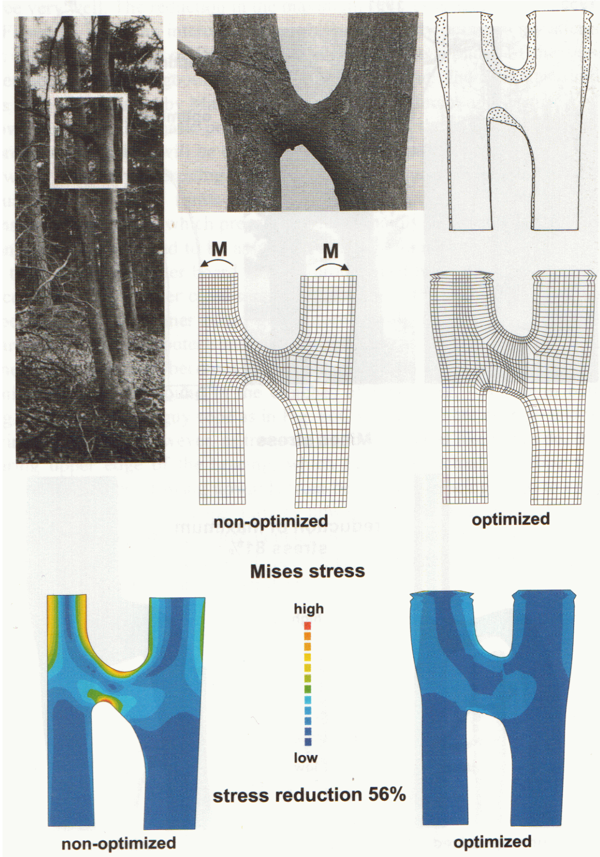


Figure 8 : Tree Framework /1/

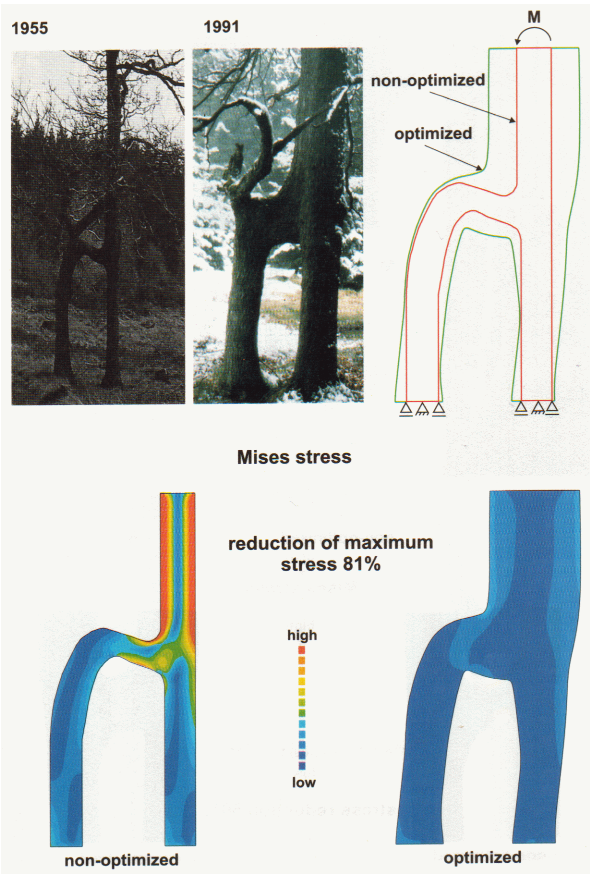


Figure 9 : Tree framework and the development over time /1/

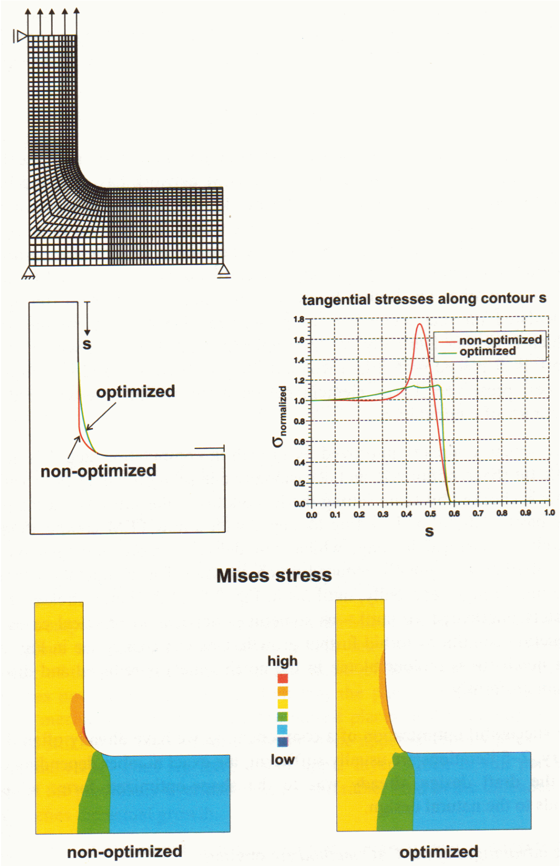
Without scientific proof but with an obvious improvement in performance the examples shown in fig. 7ff give an idea of how Mattheck found his analogy for the technical application in Optimization of mechanical structures.

As he states himself there is no mathematical or physical theory behind the proposed procedure as is with the parametric strategies, which in general fulfil mathematical and physical laws. The minimization of the cost function and their respective constituting equations can be proved but for his approach Mattheck only encourages scientist and engineers to give proof that it does not work /1/.

To show the simplicity behind this approach, that can easily be added to an existing FE program with the capability to calculate deflections due to thermal nodal or elemental load and performing a coordinate update, another typical example is shown in fig. 10f .

Using the BASIC-Scripting Language within the commercial Pre- and Postprocessor FEMAP, the results from the FE analysis were accessed, evaluated and the necessary modification from fig. 4 performed.

The result of this analysis was a stress reduction of 17% in only 2 iterations !! The same structural problem was also analysed using planar elements as in /1/ and are shown in fig. 12f.



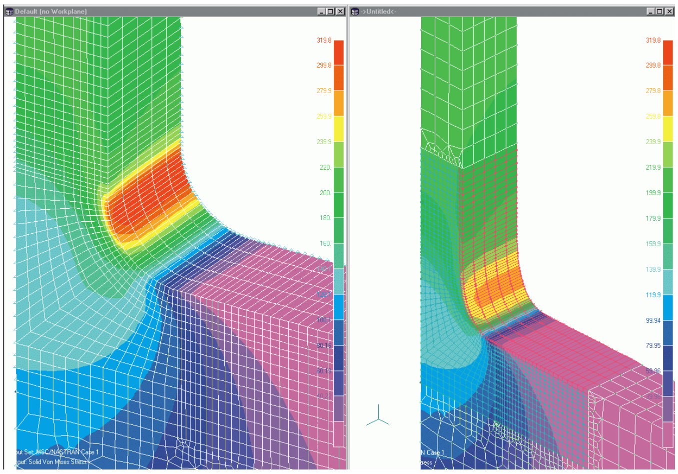
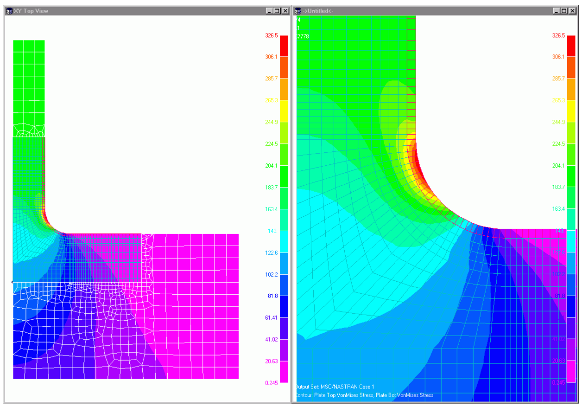


Figure 10 : Tension Plate with Narrowing Cross-Section /1/

Figure 11 : Tension Plate modelled with Solids /3/



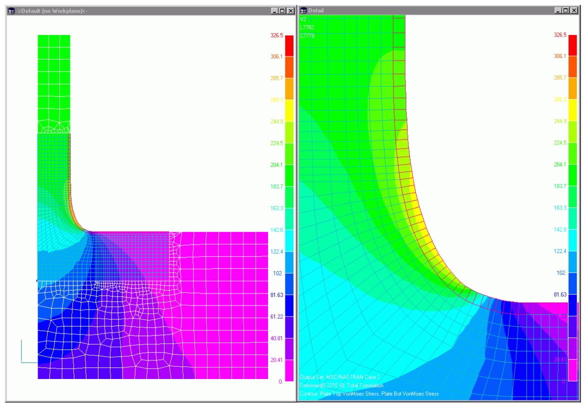


Figure 12 : Initial Notch - 326.5N/mm^2 /3/

Figure 13 : 3. Iteration - 261.3 N/mm^2 /3/

For the use of the CAO, any element type can be used providing the necessary information and capabilities. Therefore any existing FE model, planar or solid or any combination can be used.

In the FEMAP implementation of the CAO the domain under consideration for the optimization is chosen from any model (fig. 14, blue and red elements) and the growth layer (*cambium*) is defined (fig.14, red elements) thereafter.

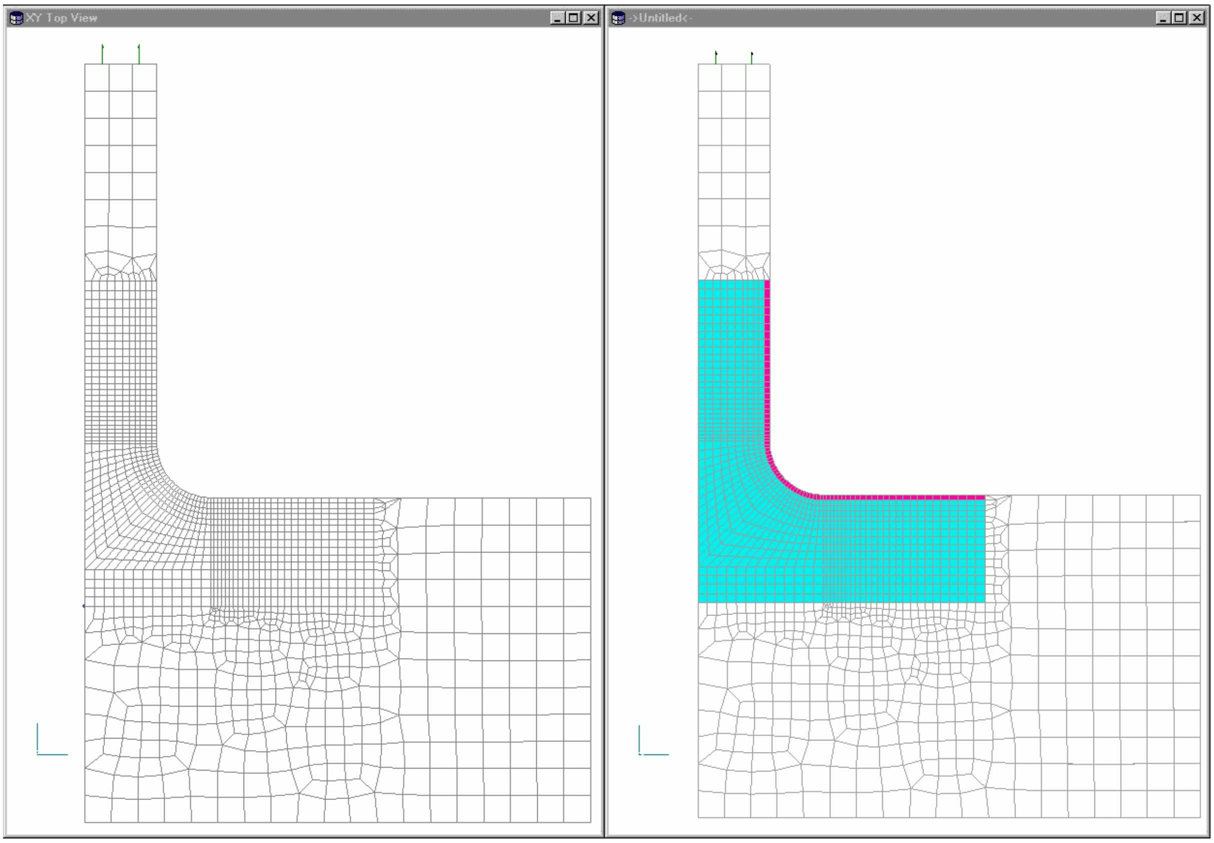


Figure 14 : FEMAP implementation of interactive selection of domain and 'cambium' elements /3/

After this, only parameters for the iteration process such as stress limits and factors for the coordinate update have to be added. The program then performs all the necessary procedures for evaluation, update, and starts the analysis program after each geometry update. Within a few minutes the results in fig. 11ff were achieved.

In /1/ only the coordinate update of the surface nodes is proposed and the possibility to update the inner nodes is not stressed out. However, to ensure a consistent quality of the element's geometry and to avoid screwed elements and elements with high aspect ratios the also proposed load case using the deflections obtained in the thermal analysis as enforced displacements should be performed. Developers of commercial programs in optimization technology tend to avoid this additional analysis by defining normals along the free edges and calculate relative displacements in the inner structure. Since it is not easy to define how to move those inner nodes in a general sense, for reasons of simplicity the enforced displacement load case is worthwhile considering.

To reference the described method to the importance in the engineering design process fig. 15f show two examples of comparing non-optimized and optimized structures under cyclic loading. Since there is an over-proportional relationship between stress reduction and the number of possible cyclic loadings the effects on the durability is impressing.

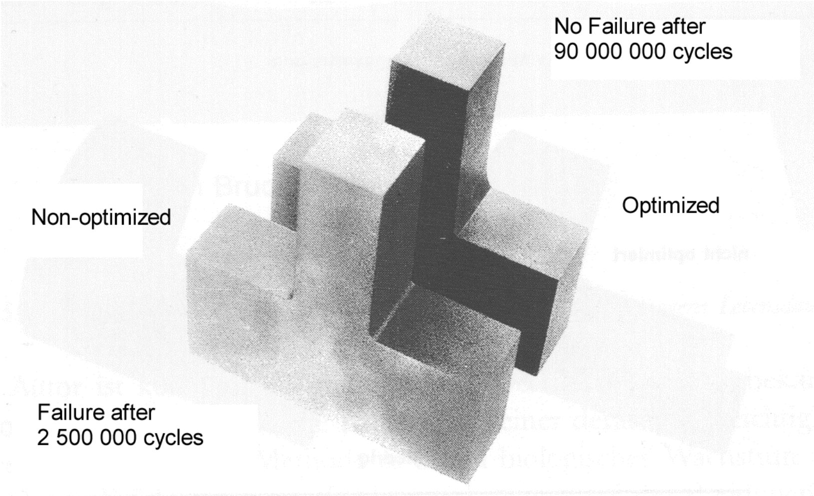


Figure 15 : Tension Plate from fig. 10 under cyclic loading /1/

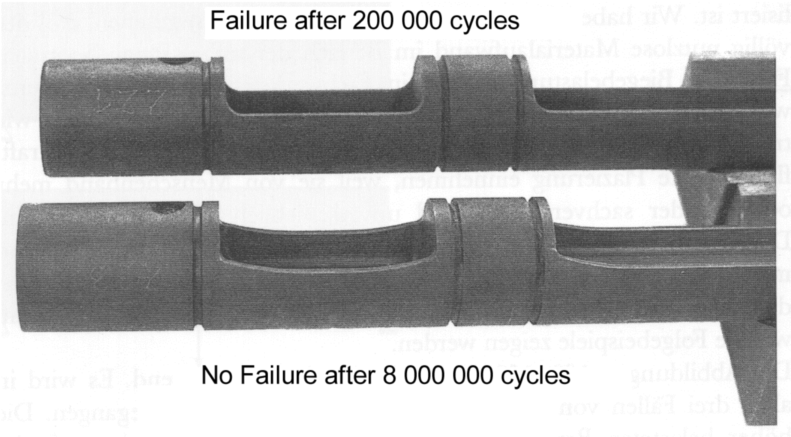


Figure 16 : Component having a life over 40 times longer with CAO /1/

Only minor changes in the contours may have high impact on the stresses and durability (see fig. 17).

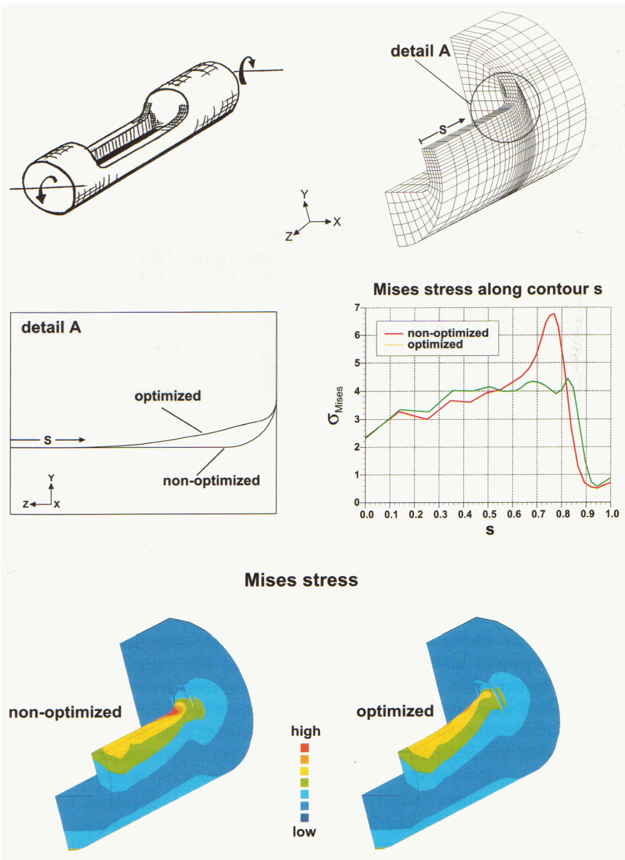


Figure 17 : Component from fig. 16 under bending before and after CAO /1/

SKO – Soft Kill Option (Away with the Ballast !)

The approach Mattheck introduces for the topological optimization of a structure to get an automatic creation of the draft design is also simple in its principles.

If you define a design domain in which your component shall be placed and apply 'outer' boundary conditions and loads to bear, this configuration is analysed and the resulting stresses are determined. The idea now is to reward those elements that carry the most of the loads by strengthening them and to 'punish' those that are lazy by weakening them. If this procedure is carefully applied and the structure is again analysed the ones that contribute most to carry the load will become stronger and stronger, whereas the 'lazy' ones will get weaker and weaker and withdraw themselves from any contribution. If they do not significantly contribute anymore they are *killed* - or better – excluded from the structure.

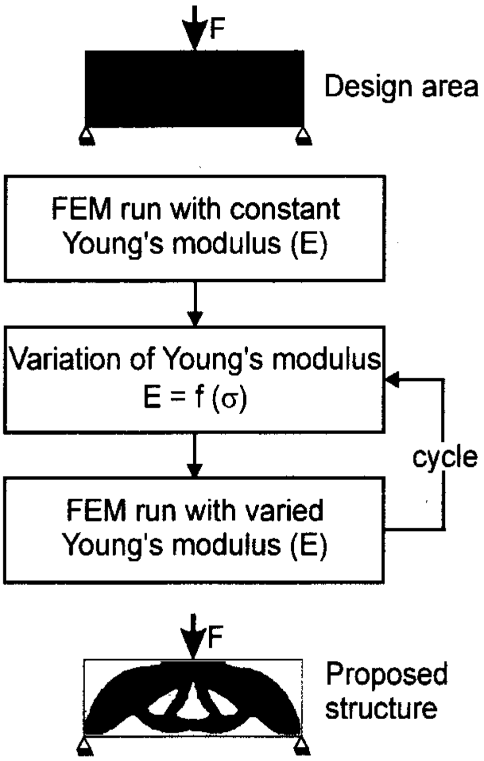


Figure 18 : SKO strategy /1/

To implement this procedure Mattheck proposes to change the Young's modulus of the individual elements according to their respective centroidal stresses.

This change can be formulated in reference to the maximum stresses in the whole structure, the increase in stress of each element according to the magnitude in the previous iteration step or as an increase in relation to the local stress with reference to the maximum global stress (see fig. 19).

In each case portions of the structure will become 'weaker' and others become 'stronger'. In any case 'load paths' will develop, which show the obviously necessary elements within the structure. There is no general rule of how to undertake the analysis and many parameters have an impact of the resulting topology. If the reference stress value is low, more elements will remain if it is high fewer elements will be necessary to carry the load.

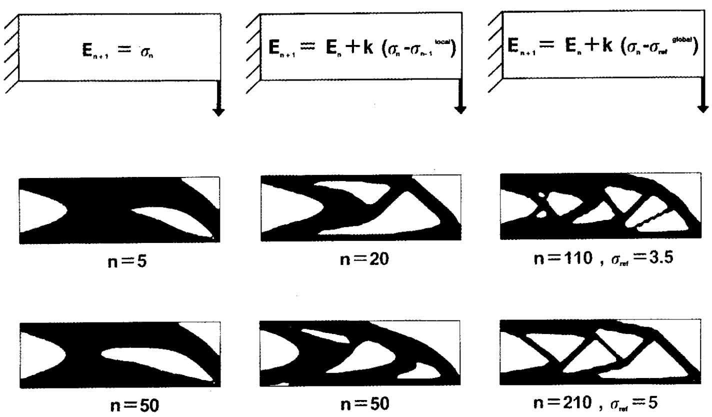


Figure 19 : Stress Method (left), Local Increments Method (middle), Global Increments Method (right) /1/

For the implementation of the SKO method, it is only necessary to evaluate the stresses at the element's centroid and to change the respective Young's modulus of this element. To better handle the number of possible moduli it is advised to have a preset number of materials to choose from and to assign elements within a certain range of stresses to one of these materials. Practice has shown that a number of about 10 materials is sufficient for a 'smooth' transition from the 'strong' to the 'weak' elements. For reasons of convergence, these materials do not have to be equally distributed (E/10, 2E/10,…, E) but can be of changing increment or decrement.

Again, the access via the BASIC scripting language in FEMAP provides all necessary functions to perform the optimization. A domain is chosen or created, constraints defined, loads applied and the program performs all necessary steps for the SKO algorithm introduced in fig. 18 . Every FE program with access to the analysis results and modification capabilities for the input data will probably work.

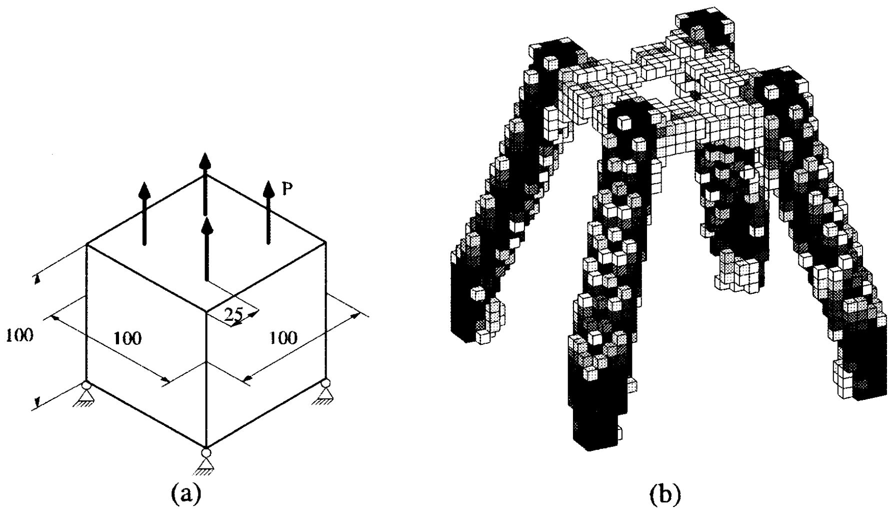
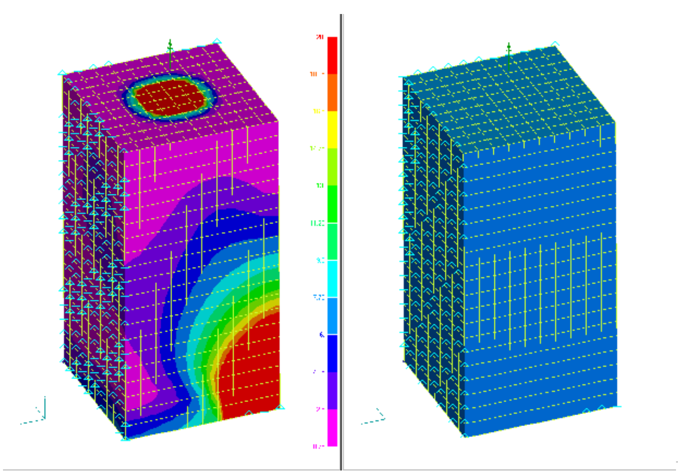


Figure 20 : Cube with top load /6/

Using the structure given in /6/ and the program developed for FEMAP the initial stress distribution is shown in fig. 21. As can be seen by comparing fig. 21 and the elements removed in fig. 22 in the first step of the iteration process all of those elements with a purple colour for their stresses were removed.



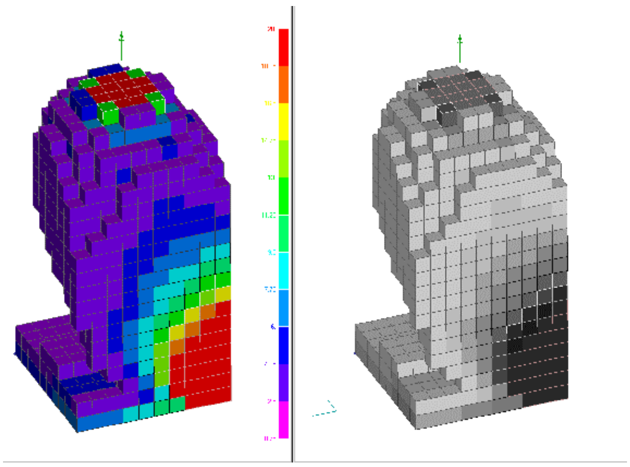


Figure 21 : ¼ model of structure in fig. 20 /3/ Figure 22 : 1. Iteration /3/

Successive weakening of the elements with low stress states results in a significantly lighter structure which obviously resembles the solution obtained in /6/ with a finer element discretization. The colour in the view to the right of the stress plot of each iteration is used to visually identify the material of the elements. The darker the colour - the stiffer the material !

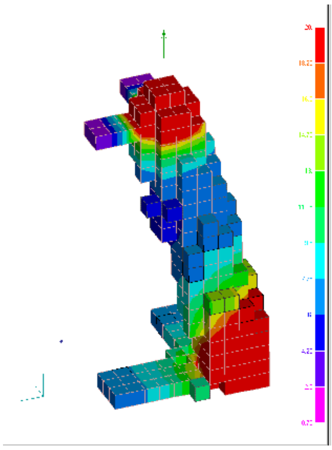


Figure 23 : Iteration 8 /3/

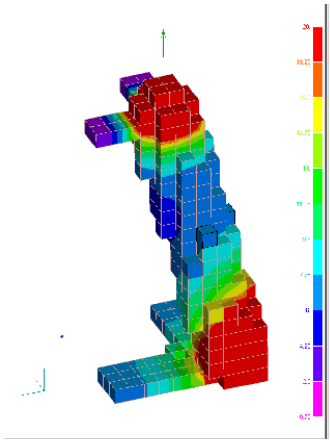


Figure 24 : Iteration 12 – Final Design Draft /3/

As can be seen in fig. 23 and fig. 24 there is no significant change in stress and material distribution between the 8th and 12th iteration and the optimum under the given parameters is achieved.

Since these design proposals are more or less ‘rough’, they need to be interpreted or ‘smoothed’. There are currently different developments for the transfer and ‘smoothing’ of these surfaces for CAD representation. Some use tessellated surfaces that might consist of a large amount of geometric data, which is hard to handle. Recent developments tend to use the results obtained from the SKO and build NURBS (Non-Uniform Rational B-Splines) to represent the structural properties where other try to identify ‘load paths’ and apply geometric components that are simpler to describe. The latter could be bars, cubes, tubes, blocks etc. and in any case, it will be up to the designer to make the final layout. Only in the area where aluminium or steel cast is used a direct use of the irregular surfaces might be possible.

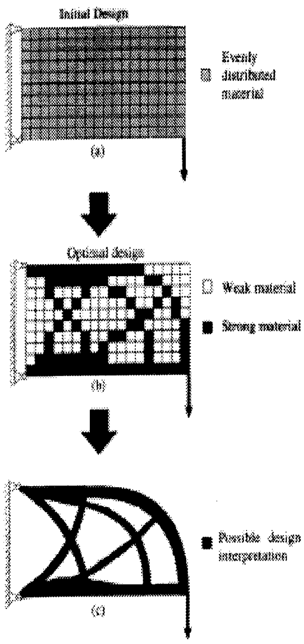
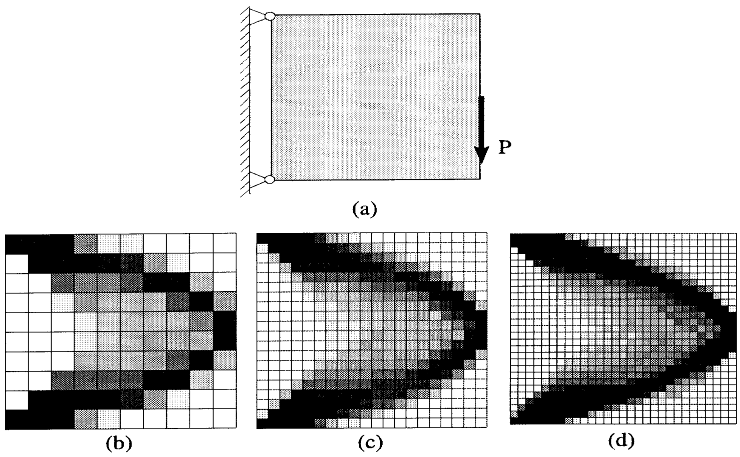


Figure 25 : Initial Design to Design Proposal and Results with different mesh densities /6/



Good Practice

In addition to the references made in the text and the book /1/ some advises should be considered in the process of using CAO and SKO as presented for optimization.

- There should be a preference for the use of 4- and 8-noded planar and 8- or 20-noded solid elements of nearly quadratic or cubic shape. 3-noded shell elements and Tetrader solids have severe problems in representing high stress gradients. Therefore, their use should be restricted to areas of lower gradients since the magnitude and the distribution is of special interest in the evaluation procedures used for CAO and SKO.

- Generally, Von Mises stresses are used in the optimization process because they are signless. There are other failure criteria possible, but Von Mises stresses are a good choice for the procedures performed for evaluation purposes.

- If no experiences exist which of the presented SKO methods should be used, the Global Incremental Method is a good choice. It will lead to a good convergence and an overall lightweight structure since the elements are modified according to their 'global' role and its change over the iteration process.

- With reference to the material in the final draft, the materials used for the SKO procedure should be fractals of the initial or 'real' Young's modulus. That means that the 'strongest' material is always 100% of the initial material that is also used for the final production. All 'weaker' materials have a respective strength below 100% and thus conclusions about their necessity or replacement can easily be made.

- Carefully analyse the stability of the resulting structure – especially for SKO optimized structures. Unless buckling analysis or non-linear analysis based on Eigenmodes is performed within the SKO procedure, every structure obtained in Optimization Technology should be carefully reviewed and imperfections in manufacturing and deviation of constraint conditions as well as loads should be considered. Remember that the methods can only optimise to what was given them as criteria or parameter. There are methods that can vary the input data in a random manner such that you get a simulation of the reality. Using these techniques in conjunction with CAO, SKO and non-linear stability will lead you to a reliable improved design.

Conclusions

Using the methods introduced by Mattheck in /1/ Shape (CAO) and Topology (SKO) Optimization can easily be performed for isotropic materials under static loads. For the use of anisotropic material Mattheck also introduces a method called CAIO, which will result in an optimal fibre layout. This will have no shear forces since the single fibres follow the direction of the principal stresses for that load case and the constraints under consideration.
For dynamic loads that can be represented by simplified static loads (modal analysis) both methods work too. There are aspects for the use of CAO and SKO even for a harmonic or transient load analysis. Usually there is a relationship between this loading, the structure, and a resulting stress field. The latter could be used for a new proposal that will be the input to the updated dynamic analysis.

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